

Proportions in Triangles

Geometric Mean: A number, x , between a and b such that $\frac{a}{x} = \frac{x}{b}$.

ex) 3 and 10

$$\frac{3}{x} = \frac{x}{10}$$

$$x^2 = 30$$

$$x = \sqrt{30} \leftarrow \text{Geometric mean.}$$

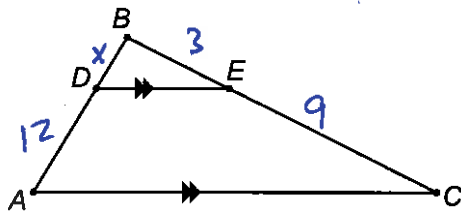
ex) 12 is geo. mean between 8 and ?

$$\frac{8}{12} = \frac{12}{b}$$

$$8b = 144$$

$$b = 18$$

Triangle "Side Splitter" Theorem: A line parallel to one side of a Δ , cuts the other 2 sides proportionally.



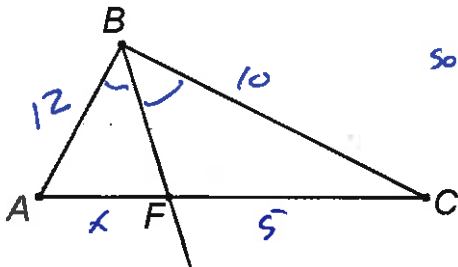
So: $\frac{BD}{DA} = \frac{BE}{EC}$ or $\frac{BD}{BA} = \frac{BE}{BC}$ or $\frac{DA}{BA} = \frac{EC}{BC}$

ex) $\frac{x}{12} = \frac{3}{9}$

$$9x = 36$$

$$x = 4$$

Triangle "Angle Splitter" Theorem: If an angle of a Δ is bisected then the opposite side is cut proportionally to the other 2 sides.



So: $\frac{AF}{AB} = \frac{FC}{BC}$

ex) Find x

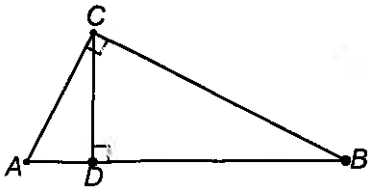
$$\frac{x}{12} = \frac{5}{10}$$

$$10x = 60$$

$$x = 6$$

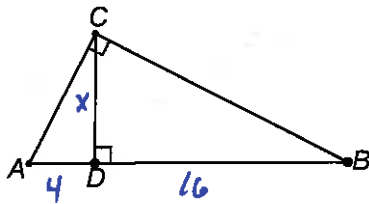
Proportions in Right Triangles

"Right Triangle Splitter" Theorem: the altitude to the hypotenuse of Right \triangle splits the \triangle into 2 Right \triangle 's each similar to the original \triangle .



so: $\triangle ADC \sim \triangle ACB$ (small \triangle \sim large \triangle)
 $\triangle CDB \sim \triangle ACB$ (medium \triangle \sim large \triangle)

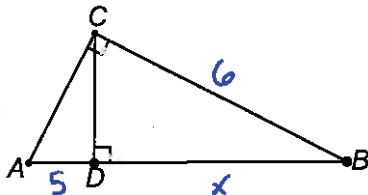
"Altitude Rule": Altitude ~~is~~ is the geometric mean between the 2 parts of the hypotenuse of the Right \triangle .



So, $\frac{AD}{CD} = \frac{CD}{DB}$
part. $\frac{AD}{CD} = \frac{CD}{DB}$
altitude = mean
part

ex) find x. $\frac{4}{x} = \frac{x}{16}$
 $64 = x^2$
 $x = 8$

"Leg Rule": Leg is the geometric mean between the whole hypotenuse and the part of the hypotenuse touching the leg of the Right \triangle .



So, $\frac{DB}{CB} = \frac{CB}{AB}$
part. $\frac{DB}{CB} = \frac{CB}{AB}$
leg = mean
whole hypot.

ex) find x:
 $\frac{x}{6} = \frac{6}{5+x}$
part $\frac{x}{6} = \frac{6}{5+x}$
mean
whole

$36 = x(5+x)$
 $36 = x^2 + 5x$
 $0 = x^2 + 5x - 36$
 $0 = (x+9)(x-4)$
 $x = -9$ or $x = 4$

36's
 18, 2
 12, 3
 9, 4 = 5
 6, 6